Quiz 4.1: Sample Answers

1. Find the derivative of $f(x) = 5\sin(x) + 4\tan(x)$.

   The derivative of $\sin(x)$ is $\cos(x)$, and $\tan(x)$ is $\sec^2(x)$, so

$$\begin{aligned} f'(x) &= 5\cos(x) + 4\sec^2(x) \\ &= 5\cos(x) + 4[1+\tan^2(x)] \text{ (a trigonometric identity)} \\ &= 5\cos(x) + 4 + 4\tan^2(x) \end{aligned}$$

2. Find the derivative of $f(x) = \cot(x) + 2\csc(x)$.

   The derivative of $\cot(x)$ is $-\csc^2(x)$, and $\csc(x)$ is $-\csc(x)\cot(x)$, so

$$\begin{aligned} f'(x) &= -\csc^2(x) + 2(-\csc(x)\cot(x)) \\ &= -(1+\cot^2(x)) - 2\csc(x)\cot(x) \text{ (a trigonometric identity)} \\ &= -1 - \cot^2(x) - 2\csc(x)\cot(x) \end{aligned}$$

3. Find the derivative of $f(x) = \frac{1+\sin x}{-x-\cos x}$.

   Using quotient rule, we have:

$$\begin{aligned} f'(x) &= \frac{(-x-\cos x)(\cos x) - (-1+\sin x)(1+\sin x)}{(-x-\cos x)^2} \\ &= \frac{-x\cos x - \cos^2 x + 1 - \sin^2 x}{(-x-\cos x)^2} \\ &= \frac{-x\cos x - 1 + 1}{(-x-\cos x)^2} \text{ (since } \sin^2 x + \cos^2 x = 1) \\ &= \frac{-x\cos x}{(-x-\cos x)^2} \end{aligned}$$

4. Find the derivative of $f(x) = 4\csc x(-2x^2 + \cot x)$.

   Using product rule, we have:

$$\begin{aligned} f'(x) &= 4\csc x(-4x - (-\csc^2 x)) + (-4\csc x\cot x)(-2x^2 - \cot x) \\ &= -16x\csc x + 4\csc^3 x + 8x^2\csc x\cot x + 4\csc x\cot^2 x \end{aligned}$$

We can then convert these in terms of $\cos x$ and $\sin x$:

$$f'(x) = \frac{16x}{\sin x} + \frac{4}{\sin^3 x} + 8x^2 \left(\frac{1}{\sin x}\right)\left(\frac{\cos x}{\sin x}\right) + 4\left(\frac{1}{\sin x}\right)\left(\frac{\cos^2 x}{\sin^2 x}\right)$$

Then we find a common denominator, $\sin^3 x$:

$$\begin{aligned} f'(x) &= \frac{16x\sin^2 x}{\sin^3 x} + \frac{4}{\sin^3 x} + \frac{8x^2 \sin x \cos x}{\sin^3 x} + \frac{4\cos^2 x}{\sin^3 x} \\ &= \frac{16x\sin^2 x + 4 + 8x^2 \sin x \cos x + 4\cos^2 x}{\sin^3 x} \end{aligned}$$